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A Sequence–Filter Joint Optimization

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***Abstract*—This paper deals with the optimisation of a sequence and its associated mismatched filter. This question has already been addressed in the literature, by alternatively solving two minimisation problem, one per sequence, meaning that the optimisation is never performed on both sequences at the same time. So, this article introduces some methods in order to optimise jointly (*i.e.* simultaneously) a sequence and its filter. First, a gradient descent is basically applied on both sequences. Second, another algorithm studies an objective function that includes the optimal mismatched filter that minimises the Integrated Sidelobe Level (or the Peak-to-Sidelobe Level Ratio). Simulations show promising results in terms of sidelobes: as expected, a joint optimisation seems to perform better than a separate one. As both methods behave differently, their choice will depend on the applications.**

***Index Terms*—Gradient descent, mismatched filter, optimisation methods, waveform design.**

I. Introduction

In radar processing, the received signal is often correlated with a filter to detect the presence of a target. If this filter is the transmitted signal, it is identified as a matched filter, and otherwise as a mismatched filter.

The matched filter is known for maximizing the SNR (Signal-to-Noise Ratio) in Gaussian Noise, but it may suffer from high sidelobes. These sidelobes can be reduced with a mismatched filter, at a cost of some loss in processing gain. In both cases, their design can be done considering an optimisation problem, in which the cost function is a criterion such as the SINR (Signal-to-Interference plus Noise Ratio), the ISL (Integrated Sidelobe Level) or the PSLR (Peak-to-Sidelobe Level Ratio).

optimising the matched filter output is a quite complicated task, especially when the transmitted sequence is subject to the constant modulus constraint, in which case the optimisation problem is not convex. Thus, a lot of methods have been reviewed in the literature [1]. Stochastic methods are usually helpful, because their convergence is *almost certain* theoretically (in the sense that, one day, a basic stochastic algorithm will converge to the global optimum) [2]. But due to the large dimension of the variables to optimise, they might not be appropriate here. Among deterministic algorithms, the gradient descent has shown interesting capabilities in similar problems [3]. Even if it converges to a local minimum — or to a saddle point — solutions obtained with this strategy present remarkable sidelobes.

On the other hand, a mismatched filter is not subject to the constant modulus constraint, so that the corresponding optimisation problem is much simpler. It has indeed been shown in [4] and in [5] that an optimal mismatched filter that minimises the ISL or the PSLR can be calculated through a convex problem. For instance, in [4], the mismatched optimisation problem is reformulated as an equivalent convex quadratically constrained quadratic program (QCQP). It may thus be possible to find a global minimum, so-to-speak an optimal mismatched filter for a given sequence.

More recently, several approaches have been introduced in order to optimise the pair transmitted sequence/mismatched filter in a sequential way, alternatively solving two optimisation problems, one per sequence. For instance, Karbasi *et al.* [6] considers a semi-definite programming (SDP) relaxation of the constraint. While in [5], a cyclic algorithm is used to study the mean-square error of the estimation of the SINR in the frequency domain.

However, all these algorithms are cyclic, meaning that the optimisation is never performed on both sequences at the same time. Hence, this article deals with a joint design of a sequence and its associated mismatched filter. Two methods are proposed here. The first method corresponds to a classic gradient descent, that will operate on both sequences at the same time, like in [7]. However, constraints on the loss-in-processing gain (due to the usage of a mismatched filter) are introduced as a pattern. The second proposed algorithm exploits the existence of the optimal mismatched filter in the ISL or the PSLR sense. This optimal solution will be inserted into the objective function, producing a signal-only optimisation problem. This new cost function will also be minimised with a gradient descent.

This article is organised as follows. Section II gives some reminders on the matched filter and the mismatched filter. In Section III, two methods are introduced for optimising jointly a sequence and its associated mismatched filter: a direct solution, based on a gradient descent, and an indirect one that exploits the existence of an optimal mismatched filter under a criterion. Some results are presented in Section IV, according to different initialisations.

Notation: In the following, bold letters designate matrices and vectors. $(.)^*$ and $(.)^H$ denote the conjugate and the transpose conjugate operator, respectively. $\circ$ and $*$ designate the Hadamard and the convolution product. $(.)^r$ denotes the reverse operator: for a given vector $\boldsymbol{a} := [a_1, \ldots, a_N]$, $\boldsymbol{a}^r := [a_N, \ldots, a_1]$. $\|.\|_2$ stands for the Frobenius norm. For an $m \times n$ matrix $\boldsymbol{A}$, it is defined by $\|\boldsymbol{A}\|_2^2 = \sum_{i=1}^m \sum_{j=1}^n |a_{i,j}|^2$.

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While $\|\boldsymbol{A}\|_\infty = \max_{i,j} |a_{i,j}|$.

II. Reminder on the Matched Filter and the Mismatched Filter

This section reminds some definitions on the matched filter and the mismatched filter. It also explains how an optimal mismatched filter can be obtained through an optimisation problem, for a given sequence. More details are given in [4].

A. Definitions

Let $\boldsymbol{s}$ be a discrete signal containing N samples:

$$\boldsymbol{s} = [s_1, s_2, \dots, s_N]^T. \tag{1}$$

In the following developments, sequence $\boldsymbol{s}$ belongs to the N-dimensional torus (or hypertorus), denoted $\boldsymbol{T}^N$, which is a non-convex set. In other words, $\boldsymbol{s}$ is subject to the constant modulus constraint. Let $\alpha_k \in [0, 2\pi[$ be the phase angle of the element s_k:

$$s_k = \begin{cases} \exp(j\alpha_k)/\sqrt{N} & \text{if } k \in [\![1, N]\!] \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Matched filtering consists in a comparison of the signal $\boldsymbol{s}$ and a time-shifted version of itself, *i.e.,* generating sequence $\boldsymbol{y}$ of length $2N-1$ such that:

$$\boldsymbol{y} = \boldsymbol{\Lambda}_N(\boldsymbol{s})\boldsymbol{s}^*, \tag{3}$$

where $\boldsymbol{\Lambda}_K(\boldsymbol{s})$ is a matrix of size $K+N-1 \times K$ containing delayed versions of the sequence $\boldsymbol{s}$, such that:

$$\boldsymbol{\Lambda}_K(\boldsymbol{s}) := \begin{bmatrix} s_N & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & s_N & \ddots & & & & \vdots \\ s_2 & & \ddots & 0 & & & \vdots \\ s_1 & s_2 & \cdots & s_N & 0 & \cdots & 0 \\ 0 & s_1 & \ddots & \vdots & s_N & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_2 & & \ddots & 0 \\ 0 & \cdots & 0 & s_1 & s_2 & & s_N \\ \vdots & & & 0 & s_1 & \ddots & \vdots \\ \vdots & & & & \ddots & \ddots & s_2 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & s_1 \end{bmatrix}. \tag{4}$$

On the other hand, processing signal $\boldsymbol{s}$ with a different filter $\boldsymbol{q}$ of length K is called mismatched filtering:

$$\boldsymbol{y} = \boldsymbol{\Lambda}_K(\boldsymbol{s})\boldsymbol{q}. \tag{5}$$

A mismatched filter $\boldsymbol{q}$ is less constrained than the matched filter, since it can take any value in $\mathbb{C}^K$. Moreover, its length may also differ from N, and can be in particular chosen to contain more elements, thus providing additional degrees of freedom compared to the matched filter. It will be assumed without loss of generality that $K = N + 2p$, $p \in \mathbb{N}$, so that the length of $\boldsymbol{y}$ is odd.

The matched filter is known for maximizing the SNR (*Signal-to-Noise Ratio*) at the peak response. Using a mismatched filter implies inevitably a loss-in-processing gain (LPG), expressed by (under a white noise hypothesis):

$$\begin{aligned} \text{LPG} &= 10\log_{10}\left(\frac{\text{SNR}_{\text{mismatched}}}{\text{SNR}_{\text{matched}}}\right) \\ &= 10\log_{10}\left(\frac{|\boldsymbol{q}^H\boldsymbol{s}|^2}{(\boldsymbol{q}^H\boldsymbol{q})(\boldsymbol{s}^H\boldsymbol{s})}\right) \leq 0. \end{aligned} \tag{6}$$

This loss-in-processing gain can be inserted as a convex constraint in optimisation problems, depending on $\boldsymbol{s}$ and $\boldsymbol{q}$, as noticed in [4]:

$$\text{LPG} \geq 10\log_{10}(\alpha) \implies \boldsymbol{q}^H\boldsymbol{q} \leq \alpha \boldsymbol{s}^H\boldsymbol{s}. \tag{7}$$

B. Optimal Mismatched Filters

Several criteria have been introduced in order to measure the performance of these filters: the Merit Factor [8], the Integrated Sidelobe Level (ISL) or the Peak-to-Sidelobe Level Ratio (PSLR).

1) Integrated Sidelobe Level: The ISL is defined by:

$$\begin{aligned} &\boldsymbol{T}^N \times \mathbb{C}^K \to \mathbb{R}+ \\ &\quad (\boldsymbol{s}, \boldsymbol{q}) \mapsto \text{ISL}(\boldsymbol{s}, \boldsymbol{q}) := \boldsymbol{y}^H\boldsymbol{F}\boldsymbol{y}, \end{aligned} \tag{8}$$

where $\boldsymbol{y}$ is defined as in (5) and $\boldsymbol{F}$ is a diagonal matrix of order $K+N-1$, defined by the vector $[1, \dots, 1, 0, 1, \dots, 1]$, with ones except for a 0 at the entry $N+p$.

The ISL can be considered as an objective function of an optimisation problem, called in this article (P_{ISL}):

$$(P_{\text{ISL}}) \begin{cases} \min_{\boldsymbol{q}} & \text{ISL}(\boldsymbol{s}, \boldsymbol{q}) \\ \text{s.t.} & \boldsymbol{s}^H\boldsymbol{q} = \boldsymbol{s}^H\boldsymbol{s}, \end{cases} \tag{9}$$

that can be solved analytically, using Lagrangian multipliers [9]:

$$\boldsymbol{q}_{\text{ISL}}(\boldsymbol{s}) = \frac{(\boldsymbol{s}^H\boldsymbol{s})\left(\boldsymbol{\Lambda}_K(\boldsymbol{s})^H\boldsymbol{F}\boldsymbol{\Lambda}_K(\boldsymbol{s})\right)^{-1}\boldsymbol{s}}{\boldsymbol{s}^H\left(\boldsymbol{\Lambda}_K(\boldsymbol{s})^H\boldsymbol{F}\boldsymbol{\Lambda}_K(\boldsymbol{s})\right)^{-1}\boldsymbol{s}}. \tag{10}$$

This last definition means that, for a given sequence $\boldsymbol{s}$, there exists a unique and optimal filter $\boldsymbol{q}_{\text{ISL}}(\boldsymbol{s})$ that minimises the ISL. Note here that this analytic solution does not guarantee an acceptable loss-in-processing gain. If the LPG is added as a second constraint in the constrained optimisation problem (P_{ISL}), then the new problem cannot be solved analytically anymore to our knowledge.

2) Peak-to-Sidelobe Level Ratio: The PSLR is expressed by:

$$\begin{aligned} &\boldsymbol{T}^N \times \mathbb{C}^K \to \mathbb{R}+ \\ &\quad (\boldsymbol{s}, \boldsymbol{q}) \mapsto \text{PSLR}(\boldsymbol{s}, \boldsymbol{q}) := \frac{\|\boldsymbol{F}\boldsymbol{y}\|_\infty^2}{|\boldsymbol{s}^H\boldsymbol{q}|}. \end{aligned} \tag{11}$$

Similarly, consider the following optimisation problem:

$$\begin{cases} \min_{\boldsymbol{q}} & \text{PSLR}(\boldsymbol{s}, \boldsymbol{q}) \\ \text{s.t.} & \boldsymbol{s}^H\boldsymbol{q} = \boldsymbol{s}^H\boldsymbol{s}. \end{cases} \tag{12}$$

This optimisation problem is equivalent to the following QCQP (*Quadratically Constrained Quadratic Program*) [4]:

$$(P_{\text{PSLR}}) \begin{cases} \min_{\boldsymbol{q},t} & t \\ \text{s.t.} & \boldsymbol{s}^H\boldsymbol{q} = \boldsymbol{s}^H\boldsymbol{s} \\ & \boldsymbol{q}^H\boldsymbol{\lambda}_{N+p+i}^H(\boldsymbol{s})\boldsymbol{\lambda}_{N+p+i}(\boldsymbol{s})\boldsymbol{q} \leq t \\ & \quad \text{for } |i| \geq 1, \end{cases} \tag{13}$$

where $\boldsymbol{\lambda}_k(\boldsymbol{s})$ is the k-th row of the matrix $\boldsymbol{\Lambda}_K(\boldsymbol{s})$. The previous optimisation problem is convex, because the cost function and all the constraints are. Hence, a global solution can be computed, denoted here by $\boldsymbol{q}_{\text{PSLR}}(\boldsymbol{s})$, using any convex optimisation method such as the interior point method for instance [10]. Note here that since the constraint on the LPG is quadratic and convex, it can easily be added to the optimisation problem (P_{PSLR}), the resulting problem still providing a global solution.

III. Joint Optimisation Algorithms

The previous section pointed out that, for a given sequence, there exists an optimal mismatched filter, whatever norm is considered. As mentioned in the introduction, some papers (see [5], [6] for instance) have dealt with an optimisation of the sequence and its associated mismatched filter, but in a sequential way: two optimisation problems are considered, one for each sequence (while the other is set as a constant), see Table I. It enables to apply the property of existence of an optimal mismatched filter, but it does not imply at all that the obtained pair is optimal.

Hence, in this section, two algorithms are introduced in order to optimise jointly a sequence and its associated mismatched filter. First, a gradient descent is applied on both sequences, on a cost function that includes a pattern. This pattern allows to control in particular the loss-in-processing gain and the sidelobe level. Second, a modified version of the cost function is considered. It exploits the existence of an optimal mismatched filter, in the PSLR or the ISL sense. A constraint can be added in order to adjust the LPG.

A. Direct Joint Optimisation

This section proposes to jointly optimise a transmitted sequence and its associated mismatched filter, via a gradient descent on $\boldsymbol{s}$ and $\boldsymbol{q}$. A similar method has already been proposed in [7] using the PSLR as a cost function: however, constraints are managed through some penalty coefficients that have to be determined empirically. Here, the sidelobe level and the loss-in-processing gain are handled through a user-defined objective pattern, inserted in the cost function. It seems reasonable to put these sidelobes as low as possible but, in practice, the convergence speed is affected. It is in fact more reliable to consider a feasible pattern an algorithm can reached, and then adjust the former (*e.g.,* a reduction of the sidelobe level) if necessary.

As said in Section II, for a sequence $\boldsymbol{s}$ of length N and its associated mismatched filter $\boldsymbol{q}$ of length $N+2p$, the processing output is of length $2N+2p-1$. And so should be the pattern, here denoted by $\boldsymbol{g}$. An example of pattern is given in Fig. 1.

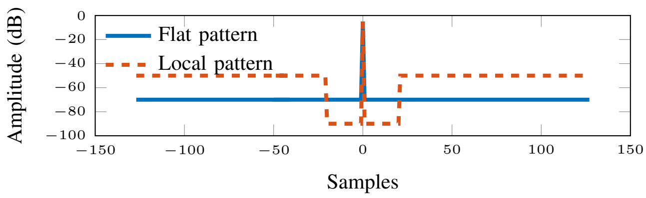

Figure 1. Some patterns. An amplified loss of 5 dB is authorized.

The modified optimisation problem, that includes the pattern, is the following:

$$\begin{cases} \min\limits_{\boldsymbol{s},\boldsymbol{q}} E_1(\boldsymbol{s},\boldsymbol{q}) := \sum\limits_{k=1}^{2N+2p-1} \left(|y_k/g_k|^{2\rho} - 1\right)^2, \\ \boldsymbol{s} \in \boldsymbol{T}^N,\ \boldsymbol{q} \in \boldsymbol{C}^K, \end{cases} \tag{14}$$

and where $\boldsymbol{y}$ is defined as in Section II. This optimisation problem searches for couple $(\boldsymbol{s},\boldsymbol{q})$ that provides the closest output to the pattern.

In order to apply a gradient descent, a gradient computation of the cost function is needed, which can be done numerically with finite differences. Otherwise, for sequence $\boldsymbol{s}$, notice that the considered partial derivatives are computed with respect to the phase α_k of each element s_k, so that the constant modulus property is respected:

$$\frac{\partial E_1}{\partial \alpha_k} = -2\rho\Im[s_k\left((\boldsymbol{\gamma}\circ\boldsymbol{y}) * \boldsymbol{q}\right)_k], \qquad k \in [\![1,N]\!] \tag{15}$$

where $\gamma_k = \left(|y_k/g_k|^{2\rho} - 1\right)|y_k/g_k|^{2\rho-1}\left(1/g_k^2\right)$. While for the mismatched filter, Wirtinger derivatives are used:

$$\frac{\partial E_1}{\partial q_k} = \rho\left((\boldsymbol{\gamma}\circ\boldsymbol{y}) * \boldsymbol{s}^r\right), \qquad k \in [\![1,2N+2p-1]\!] \tag{16}$$

After convergence of the steepest descent step, the pattern may be slightly adjusted (reduction of 1 dB of the sidelobes for instance). The whole procedure is summarized in Table II.

B. Indirect Joint Optimisation

An iterative algorithm is also proposed in this section. It takes advantage of the existence of an optimal mismatched filter that minimises the PSLR or the ISL, as explained in Section II. Only the latter is detailed in this section, even though the following procedure can easily be extended for the PSLR.

For a joint optimisation, the optimisation problem (P_{ISL}) becomes:

$$(P_{\text{ISL}}^{(2)}) \begin{cases} \min\limits_{\boldsymbol{s},\boldsymbol{q}} E_2(\boldsymbol{s},\boldsymbol{q}) := \|\boldsymbol{F}\boldsymbol{\Lambda}_K(\boldsymbol{s})\boldsymbol{q}\|_2^2 \\ \text{s.t. } \boldsymbol{s}^H\boldsymbol{q} = \boldsymbol{s}^H\boldsymbol{s} \\ \boldsymbol{s} \in \boldsymbol{T}^N,\ \boldsymbol{q} \in \boldsymbol{C}^K. \end{cases} \tag{17}$$

Notice that the only difference is on the optimisation variables, with the addition of $\boldsymbol{s}$. Since sequence $\boldsymbol{s}$ belongs to an hypertorus, which is a non-convex set, this problem cannot be freely solved globally, contrary to the original problem (P_{ISL}).

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Separating both optimisation variables gives the following problem:

$$(P_{\mathrm{ISL}}^{(3)}) \begin{cases} \min_{\boldsymbol{s}} \min_{\boldsymbol{q}} E_2(\boldsymbol{s}, \boldsymbol{q}) = \|\boldsymbol{F}\boldsymbol{\Lambda}_K(\boldsymbol{s})\boldsymbol{q}\|_2^2 \\ \text{s.t. } \boldsymbol{s}^H \boldsymbol{q} = \boldsymbol{s}^H \boldsymbol{s} \\ \boldsymbol{s} \in \boldsymbol{T}^N,\ \boldsymbol{q} \in \boldsymbol{C}^K. \end{cases} \tag{18}$$

In this expression, (P_{ISL}) can be identified. As mentioned earlier, there exists a global solution that can be computed, denoted $\boldsymbol{q}_{\mathrm{ISL}}(\boldsymbol{s})$ — its explicit defintion is defined in Section II. Remark that $\boldsymbol{q}_{\mathrm{ISL}}(\boldsymbol{s})$ can also be computed through a QCQP. This solution is put into the objective function, becoming a function of a sequence-only variable:

$$(P_{\mathrm{ISL}}^{(4)}) \begin{cases} \min_{\boldsymbol{s}} E_3(\boldsymbol{s}) := E_2(\boldsymbol{s}, \boldsymbol{q}_{\mathrm{ISL}}(\boldsymbol{s})) \\ \boldsymbol{s} \in \boldsymbol{T}^N. \end{cases} \tag{19}$$

As such, the definition of the mismatched filter is therefore inherent, given the expression of cost function E_3. At the end of the day, the proposed algorithm is actually akin to a simple minimisation problem, that can be solved using a gradient descent for instance. Notice that a numerical computation of the derivative requires to consider an optimisation problem (P_{ISL}) for each phase α_k...

The algorithm is summarized in Table III. Remind that it can easily be extended to an optimisation of the PSLR criterion, and/or with constraints on the loss-in-processing gain.

IV. Results

This section presents some results obtained with the previous described algorithms. Drawn sequences are of length $N = 64$. Different initialisations are compared.

To solve all the QCQP, the CVX package has been used. CVX is a package for specifying and solving convex programs [10].

In the following results, different pairs sequence/filter have been considered, obtained through different strategies. They will denoted as follows:

- "Matched opt.": an optimisation of the matched filter has been performed, like in [3].
- "Direct opt.": a direct joint optimisation with a gradient descent, described in Section III-A.
- "Indirect opt.": an indirect joint optimisation, proposed in III-B.

A. First Results

Figure 2 illustrates the usefulness of a joint optimisation. It compares a sequential algorithm with a joint one, proposed in the previous section. Therefore, we have first considered a random sequence, optimised it in the sense of a matched filter, and then computed its optimal mismatched filter in the PSLR sense. Second we have used this sequence as the initialisation sequence for the proposed joint optimisation algorithm. Whatever method, the pair sequence/mismatched filter provided by the proposed joint optimisation algorithm outperforms the results provided by the separated optimisation, of around 10 dB.

Table I
A cyclic algorithm

Algorithm	Cyclic optimisation with a gradient descent
Given	Initial sequence $\boldsymbol{s}$
	Initial mismatched filter $\boldsymbol{q}$
Repeat	1. Gradient descent search on $\boldsymbol{s}$, $\boldsymbol{q}$ fixed (see below)
	2. Computation of $\boldsymbol{q}_{\mathrm{PSLR}}(\boldsymbol{s})$, optimal solution of (P_{PSLR})
Until	A stopping criterion is satisfied.

Table II
A direct joint optimisation algorithm

Algorithm	Direct joint optimisation with a gradient descent
Given	Initial sequence $\boldsymbol{s}$
	Initial mismatched filter $\boldsymbol{q}$
Repeat	1. Gradient descent search
	— Computation of $\nabla_{\boldsymbol{\alpha}} E_1$, $\nabla_{\boldsymbol{q}} E_1$
	— Search of the best step μ
	— Update of $\boldsymbol{s} : s_k = s_k\, \mathrm{e}^{-j\mu(\nabla_{\boldsymbol{\alpha}} E_1)_k}$, $k \in [\![1, N]\!]$
	— Update of $\boldsymbol{q} : q_k = q_k - \mu(\nabla_{\boldsymbol{q}} E_1)_k)$, $k \in [\![1, K]\!]$
	— Repeat until convergence
	2. Readjustment of the pattern $\boldsymbol{g}$
Until	A stopping criterion is satisfied.

Table III
An indirect joint optimisation algorithm

Algorithm	Indirect joint optimisation based on a gradient descent
Given	Initial sequence $\boldsymbol{s}$
Repeat	1. Gradient descent search
	— Computation of the gradient vector $\nabla_{\boldsymbol{\alpha}} E_3$
	— Search of the best step μ
	— Update of $\boldsymbol{s} : s_k = s_k\, \mathrm{e}^{-j\mu(\nabla_{\boldsymbol{\alpha}} E_3)_k}$, $k \in [\![1, N]\!]$
Until	A stopping criterion is satisfied.

A comparison of both optimisation methods described in this article is represented in Figure 3. A loss-in-processing gain of 1 dB has been allowed. In terms of PSLR, a direct gradient descent seems more appropriate (PSLR = -58 dB). However, note that this pattern-based method cannot easily been adapted to provide good ISL sequences. In that aspect, the second proposed method provides very interesting performance in terms of ISL, with very low sidelobes near the mainlobe, at the expense of relatively higher sidelobes at the end of the sequences (yet at a very acceptable level, around -40 dB, at most).

B. On the Influence of the Initialisation

It has already been mentioned that, theoretically, a gradient descent converges to the closest local minimum (at least to the closest saddle point). In this regard, the initialisation is quite important, because it influences the sidelobe level of the obtained pair sequence/filter. Two initialisations are thus compared in this section: the first one was a random initialisation (see the previous paragraph), while the second one uses as initialisation a sequence already optimised in the PSLR sense. Results are presented Figure 4.

Starting from the random sequence drawn in Section IV-A, an optimisation of its matched filter is performed, such as in [3]. The obtained sequence is used as an initialisation for both

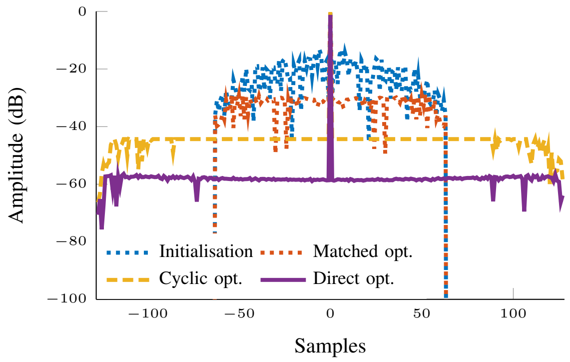

Figure 2. Separated Optimisation vs. Joint Optimisation

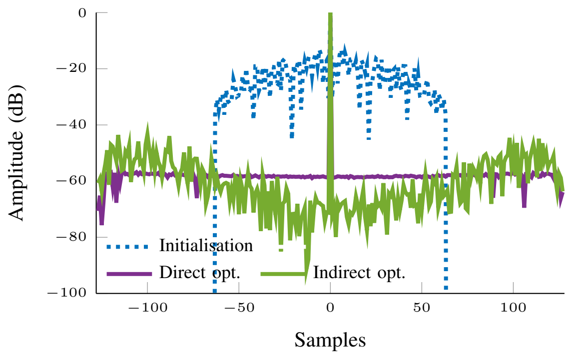

Figure 3. Comparison of Joint Optimisation Methods

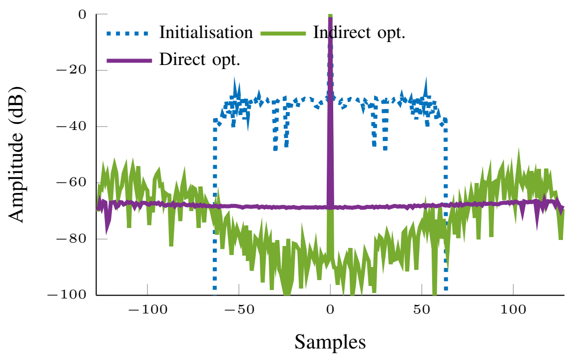

Figure 4. Joint Optimisation, Starting from a "Flat" Sequence

proposed methods. In addition to low sidelobes, it is interesting to look at their distributions, according to the method. A direct optimisation of the PSLR gives — as expected — a uniform distribution, with sidelobes at -58 dB, improved by 8 dB with the other init. On the other hand, the indirect optimisation over the ISL provides incredibly good results, with sidelobes around the mainlobe reaching levels as low as -90 dB while the sidelobes at the edges are still acceptable, around -50 dB. To our knowledge, such sidelobes level have not been reached in the literature. The particular shape obtained in this ISL case may besides be very interesting in radar applications, as it may improve the detection of close and small targets that are usually hidden in strong clutter.

V. Conclusion

In this article, several algorithms have been introduced, in order to design jointly a sequence and its mismatched filter. Unlike methods described in the literature, the optimisation process is performed simultaneously on both sequences, and not alternatively. Simulations have highlighted really promising results on the sidelobes, better than cyclic algorithms:

- In terms of sidelobe levels, obtained results are quite satisfying, even with small sequences.
- According to the algorithm, their distribution is different. A joint direct optimisation of the PSLR provides — as expected, because of the chosen pattern — a mismatched filter output with a flat profile. While an indirect joint optimisation over the ISL shows amazingly low levels around the mainlobe.

Ongoing works will be focused on:

- applying other methods instead of a gradient descent;
- the initialisation choice for the mismatched filter;
- an extension of these algorithms for a set of sequences (and their associated set of mismatched filters);
- comparing output levels with some lower bounds on the correlation sidelobe level of sets of sequences, such as Welch or Levenshtein bounds. Note that, for a single sequence, these bounds are equal to 0 ($-\infty$ dB), which leaves some possibility to reach even lower sidelobe levels with joint optimisation methods.

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